

Filter Design on a Budget

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ABSTRACT

A question that repeatedly gets asked is “what values of R and C will give a filter response closest to a given frequency?” This note gives a quick answer to that question. A second question that often is asked “does a lab really need to have a complete stock of 1% resistors in order to implement filters of any frequency?” While there is no single answer that fits every situation or budget, this note gives the most complete answer possible.

1 Introduction

The author recently changed jobs to become a Texas Instruments Application Specialist. One of the first actions taken was to set up a laboratory stock of components. The technician took one look at the list of parts requisitioned and responded with the same response heard from every technician over the years: “Are all of these part values really necessary? Can some be eliminated to get the cost down?” The answer has always been “Yes, it is necessary to be able to implement any filter frequency”. But there has always been a doubt – is that answer really true?

2 A Question of Decades

Not just units of time – but also decades of part values. Passive component values are available in decades, with part values following a logarithmic sequence within each decade. If there is a 1.2-k Ω resistor, for example, 1.2 Ω , 12 Ω , 120 Ω , 12 k Ω , 120 k Ω , 1.2 M Ω , 12 M Ω will also be available. This note can only give general guidance about what decades are needed:

- Filters scale over a fairly wide range. If a resistor is increased by a decade and the capacitor is reduced by a decade, the filter response will remain unchanged. The response of a filter that uses 100 k Ω and 0.1 μ F will be the same with 10 k Ω and 1 μ F.
- Resistors that are too low will increase power consumption in the circuit, and resistor values that are too high will increase noise.
- High-speed applications use lower values of resistors in the 100 Ω to 1 k Ω range, precision equipment operates best with resistors in the 100 k Ω to 1 M Ω range, while portable equipment uses higher values in the 100 k Ω to 10 M Ω range.
- 1% resistors below 100 Ω and above 10 M Ω are hard to obtain. Precision capacitors below 100 pF and above 0.1 μ F are hard to obtain.

A company's product line and past experience can be used to estimate how many decades are needed. Three decades should be enough. Therefore, any lab procurement will need to include sets of resistors recommended below in three decades for capacitor, and three decades for resistor.

3 The E Sequence of Component Values

Component values follow a logarithmically derived sequence of values, which repeat every decade. The E-values correspond to component tolerance values, which were used to determine the next logarithmic step. For 10% / E-12 components, for example, a component with a value of 1 can be as low as 0.9 or as high as 1.1. The next value in the sequence, with a value of 1.2, can be as low as 1.08 or as high as 1.32. Ideally, there would be no overlap between adjacent values – but there was some arbitrary rounding done originally to simplify the values.

For the E-12 sequence, there are 12 logarithmic steps, likewise 24 for E-24 and 96 for E-96. Values other than these standard values are almost always special order items, with long lead times and added expense.

3.1 E-12 Resistor and Capacitor 10% Values

1.0, 1.2, 1.5, 1.8, 2.2, 2.7, 3.3, 3.9, 4.7, 5.6, 6.8, and 8.2.

3.2 E-24 Resistor and Capacitor 5% Values (Also 1% Capacitors)

1.0, 1.1, 1.2, 1.3, 1.5, 1.6, 1.8, 2.0, 2.2, 2.4, 2.7, 3.0, 3.3, 3.6, 3.9, 4.3, 4.7, 5.1, 5.6, 6.2, 6.8, 7.5, 8.2, and 9.1.

3.3 E-96 Resistor 1% Values

1.00, 1.02, 1.05, 1.07, 1.10, 1.13, 1.15, 1.18, 1.21, 1.24, 1.27, 1.30, 1.33, 1.37, 1.40, 1.43, 1.47, 1.50, 1.54, 1.58, 1.62, 1.65, 1.69, 1.74, 1.78, 1.82, 1.87, 1.91, 1.96, 2.00, 2.05, 2.10, 2.15, 2.21, 2.26, 2.32, 2.37, 2.43, 2.49, 2.55, 2.61, 2.67, 2.74, 2.80, 2.87, 2.94, 3.01, 3.09, 3.16, 3.24, 3.32, 3.40, 3.48, 3.57, 3.65, 3.74, 3.83, 3.92, 4.02, 4.12, 4.22, 4.32, 4.42, 4.53, 4.64, 4.75, 4.87, 4.99, 5.11, 5.23, 5.36, 5.49, 5.62, 5.76, 5.90, 6.04, 6.19, 6.34, 6.49, 6.65, 6.81, 6.98, 7.15, 7.32, 7.50, 7.68, 7.87, 8.06, 8.25, 8.45, 8.66, 8.87, 9.09, 9.31, 9.53, 9.76.

3.4 Other Sequences

Other sequences exist:

- E-192 for 1/2% resistors
- E-6 for 20% resistors and capacitors – now seldom used

These other sequences are not discussed in this note – although designers attempting a three or four pole filter with a single op amp will soon become acquainted with the E-192 series, or even higher precision resistors.

4 Four Filter Precisions

Texas Instruments has recently introduced a spreadsheet that will calculate the optimum values of R and C for a given frequency. This note uses that tool. Ninety frequency values were selected from 1.0 to 9.9 (corresponding to an arbitrary decade), and the optimum R and C value for each frequency was recorded. This was repeated for four cases:

- E-96 resistors and E-24 capacitors
- E-96 resistors and E-12 capacitors
- E-24 resistors and E-24 capacitors
- E-24 resistors and E-12 capacitors

It should be noted that the author never recommends that 10% capacitors ever be used in filter design, and that critical filter designs often need 1% capacitors. When 10% is selected in the RC calculation spreadsheet, it is understood that the actual capacitors should be 5% tolerance parts, located at E-12 values.

If readers only want a table of optimum passive component values for each frequency, they can stop reading here, and print this application note for the four tables that follow. If the reader wants to procure parts, then the rest of the document will be helpful.

5 What Component Values Need to Be Procured?

Each case above was analyzed to see if all component values were really needed. In every case, certain component values appeared repeatedly, therefore being more important for a well-stocked filter design laboratory. Some good examples are:

- The E-96 value 4.42, which appears for ten frequencies in Table 1.
- The E-96 value 2.21, which appears for eleven frequencies in Table 1.

So far, so good. This accounts for a total of 21 out of 90 frequency combinations, meaning that a substantial reduction in individual part values is possible.

5.1 The E96-E24 Combination

Traditionally, 1% resistors and 5% capacitors have been the default combination of component values for precision filter design. Table 1 shows the optimum component values for 90 values of frequency. As already stated, a lot of component values are repeated. In addition, there are many cases in which two or even three combinations of components produce identical low error values. In these cases, the frequency values are simply repeated in the table for each combination. Some of these frequency combinations are highlighted orange. These are cases that would add to the number of separate E-96 values required. Because there is another way to produce these frequencies, these combinations will receive no more consideration. The remaining E-96 resistor values used to generate all 90 combinations of frequency are listed in a small section on the right side of the table. If all frequency combinations are desired, only 48 out of 96 values or resistor need to be purchased. If 22 out of 90 frequency combinations can be deleted, the number of unique resistor values required goes down to only 23 values, which would be much more economical to procure. The deleted frequencies and resistor values are highlighted in yellow.

5.2 The E96-E12 Combination

Table 2 shows the optimum component values for 90 values of frequency using 1% resistors and 10% capacitor values (the actual tolerance of purchased components should be 5%). Assuming that resistors are relatively economical compared to capacitors, eliminating every other capacitor value would result in substantial cost savings. In addition the capacitor values in between the E-12 values are difficult to obtain. Capacitor manufacturers do not like to supply these values. Applying the same analysis techniques as to the previous table, 59 unique values of 1% resistors are needed to generate all 90 values of frequency. There are 11 more resistor values than in the previous case, but it saves 12 hard to obtain capacitor values. This is a good trade-off. If a lower cost alternative is desired, then 24 unique values of resistor will allow 58 frequencies to be generated. Frequency and resistor values that are deleted are shown in yellow.

Freq	C	R	Freq	C	R	
1.0	3.6	4.42	5.2	3.0	1.02	1.00
1.1	3.6	4.02	5.2	1.2	2.55	1.02
1.2	1.3	1.00	5.3	5.6	5.36	1.07
1.2	3.9	3.40	5.4	9.1	3.24	1.13
1.2	3.0	4.42	5.5	2.7	1.07	1.15
1.3	3.6	3.40	5.6	1.8	1.58	1.18
1.4	3.6	3.16	5.6	1.2	2.37	1.21
1.5	2.4	4.42	5.7	3.0	9.31	1.30
1.6	3.9	2.55	5.8	6.2	4.42	1.37
1.7	3.6	2.61	5.9	2.7	1.00	1.40
1.7	2.7	3.48	5.9	1.8	1.50	1.43
1.8	6.8	1.30	6.0	1.2	2.21	1.54
1.8	2.0	4.42	6.1	1.5	1.74	1.58
1.9	1.2	6.98	6.1	1.0	2.61	1.62
2.0	3.6	2.21	6.1	7.5	3.48	1.87
2.0	1.8	4.42	6.2	2.4	1.07	2.00
2.1	2.4	3.16	6.3	1.6	1.58	2.15
2.2	1.8	4.02	6.4	2.2	1.13	2.21
2.3	1.6	4.32	6.4	1.1	2.26	2.26
2.4	5.1	1.30	6.5	2.4	1.02	2.37
2.4	3.0	2.21	6.6	8.2	2.94	2.43
2.4	1.5	4.42	6.7	9.1	2.61	2.49
2.5	9.1	6.98	6.8	1.8	1.30	2.55
2.6	2.4	2.55	6.9	5.6	4.12	2.61
2.6	1.8	3.40	7.0	9.1	2.49	2.94
2.7	1.1	5.36	7.1	1.6	1.40	3.09
2.8	4.7	1.21	7.2	1.0	2.21	3.16
2.9	1.1	4.99	7.3	4.7	4.64	3.24
3.0	2.4	2.21	7.4	1.0	2.15	3.40
3.0	1.2	4.42	7.5	1.8	1.18	3.57
3.1	3.6	1.43	7.6	4.3	4.87	4.02
3.2	2.2	2.26	7.7	1.8	1.15	4.12
3.3	1.2	4.02	7.8	2.0	1.02	4.22
3.4	3.6	1.30	7.9	9.1	2.21	4.32
3.5	9.1	4.99	8.0	8.2	2.43	4.42
3.6	2.0	2.21	8.1	6.2	3.16	4.64
3.6	1.0	4.42	8.2	8.2	2.37	4.75
3.7	5.6	7.68	8.3	6.2	3.09	4.87
3.8	8.2	5.11	8.4	1.2	1.58	4.99
3.9	1.6	2.55	8.5	1.0	1.87	5.11
3.9	1.2	3.40	8.6	3.9	4.75	5.36
4.0	3.9	1.02	8.7	6.2	2.94	6.34
4.0	1.8	2.21	8.8	1.6	1.13	6.49
4.1	4.7	8.25	8.9	1.1	1.62	6.98
4.2	4.7	8.06	9.0	3.3	5.36	7.68
4.3	2.7	1.37	9.1	2.7	6.49	8.06
4.4	1.6	2.26	9.2	5.6	3.09	8.25
4.5	1.6	2.21	9.3	2.7	6.34	9.31
4.6	8.2	4.22	9.4	6.8	2.49	
4.7	2.2	1.54	9.4	5.1	3.32	
4.8	1.5	2.21	9.5	2.4	6.98	
4.8	7.5	4.42	9.6	7.5	2.21	
4.9	9.1	3.57	9.7	8.2	2.00	
5.0	2.7	1.18	9.8	1.0	1.62	
5.1	2.4	1.30	9.9	3.0	5.36	

Table 1. E96-E24 Component Values

Freq	C	R	Freq	C	R	Freq	C	R
1.0	1.0	1.58	5.6	1.8	1.58	1.00	3.40	
1.1	2.7	5.36	5.6	1.2	2.37	1.02	3.48	
1.2	3.9	3.40	5.7	2.2	1.27	1.07	3.74	
1.3	1.2	1.02	5.8	1.0	2.74	1.13	4.02	
1.4	2.7	4.22	5.9	2.7	1.00	1.15	4.12	
1.5	1.8	5.90	5.9	1.8	1.5	1.18	4.22	
1.6	3.9		6.0	1.2		1.21	4.32	
1.7	2.7	3.48	6.1	1.5	1.74	1.27		
1.8	6.8	1.30	6.1	1	2.61	1.30	4.64	
1.9	1.2	6.98	6.2	2.7	9.53	1.37	4.75	
2.0	1.8	4.42	6.3	2.2	1.15	1.54	5.11	
2.1	1.8	4.22	6.4	2.2	1.13	1.58	5.23	
2.2	1.8	4.02	6.5	2.7	9.09	1.62	5.36	
2.3	1.2	5.76	6.6	8.2	2.94	1.69	5.49	
2.4	1.5		6.7	1.5		1.74	5.76	
2.5	1.0	6.34	6.7	1	2.37	1.87		
2.6	1.8	3.40	6.8	1.8	1.30	1.91	6.34	
2.7	1.0	5.90	6.9	5.6	4.12	2.00	6.49	
2.8	4.7	1.21	7.0	2.7	8.45	2.15	6.65	
2.9	1.0	5.49	7.1	1.2	1.87	2.21	6.98	
3.0	1.2	4.42	7.2	1.0	2.21	2.26	7.32	
3.1	2.7	1.91	7.3	4.7	4.64	2.43	7.50	
3.2	2.2		7.4	1.0		2.49	7.68	
3.3	1.2	4.02	7.5	1.8	1.18	2.55		
3.4	2.7	1.74	7.6	5.6	3.74	2.74	8.25	
3.4	1.8	2.61	7.7	1.8	1.15	2.87	8.45	
3.5	2.7	1.69	7.8	3.9	5.23	2.94	9.09	
3.6	1.0	4.42	7.9	2.7	7.50	3.01	9.31	
3.7	5.6	7.68	8.0	8.2	2.43	3.09	9.53	
3.8	8.2	5.11	8.1	2.7	7.32	3.24		
3.9	1.2		8.2	8.2				
4.0	3.9	1.02	8.3	1.0	1.91			
4.0	1.8	2.21	8.4	1.2	1.58			
4.1	4.7	8.25	8.5	1.0	1.87			
4.2	4.7	8.06	8.6	3.9	4.75			
4.3	2.7	1.37	8.7	1.8	1.02			
4.4	1.2	3.01	8.8	3.9	4.64			
4.5	8.2	4.32	8.9	2.7	6.65			
4.6	8.2		9.0	3.3				
4.7	2.2	1.54	9.1	2.7	6.49			
4.8	1.5	2.21	9.2	5.6	3.09			
4.9	1.0	3.24	9.3	2.7	6.34			
5.0	2.7	1.18	9.4	6.8	2.49			
5.1	4.7	6.65	9.5	1.8	9.31			
5.2	1.2	2.55	9.6	6.8	2.43			
5.3	5.6	5.36	9.7	8.2	2.00			
5.4	1.0		9.8	1.0				
5.5	2.7	1.07	9.9	5.6	2.87			

Table 2. E96-E12 Component Values

5.3 The E24-E24 and E24-E12 Combinations

The same analysis techniques were applied to the E24-E24 and E24-E12 cases in Table 3. In these cases, there is very little motivation for reducing the number of resistor values, as good resistor kits containing these values are relatively inexpensive. Combinations pair, because the resistor and capacitor values can interchange.

There is very little advantage to purchasing all E24 capacitor values, because all but 16 frequencies can be produced optimally with E12 capacitor values. In addition, close examination reveals that two or three frequencies are produced by the same combination. This is because the combinations are only accurate to 2 1/2% in some cases, causing overlap. There are really only 12 discrete frequencies produced by seven additional capacitor values. Therefore, only 19 values of capacitance and 24 of resistance are needed per decade.

The designer is cautioned that 5% components are only useable for low order (one or two pole) lowpass and highpass filters. Precision bandpass and notch filter design with anything but 1% resistors and capacitors is usually disastrous. Figure 1 shows a notch filter implemented with different tolerances of passive components. The response with perfect components is shown in green for each combination. Deep notches at other frequencies may, or may not have occurred in the random worst case examples – but the primary issue is the worst-case notch depth. With 1% resistors and 1% capacitors, the very worst-case notch depth is about 18 dB. This is almost a 10:1 reduction – well worth the effort of implementing the notch – especially since the worst case is a statistical anomaly and typical performance is much better.

The situation deteriorates rapidly with 1% resistors and 5% capacitors. The simulation program was not looking for deep notches and did not randomly find any. The notch depth at the center frequency can only be assured to be 6 dB, which is only a 2:1 rejection.

When 5% resistors and 5% capacitors are used, one of the worst-case examples hardly found a notch at all, which means there is no assurance that the notch filter will operate at all.

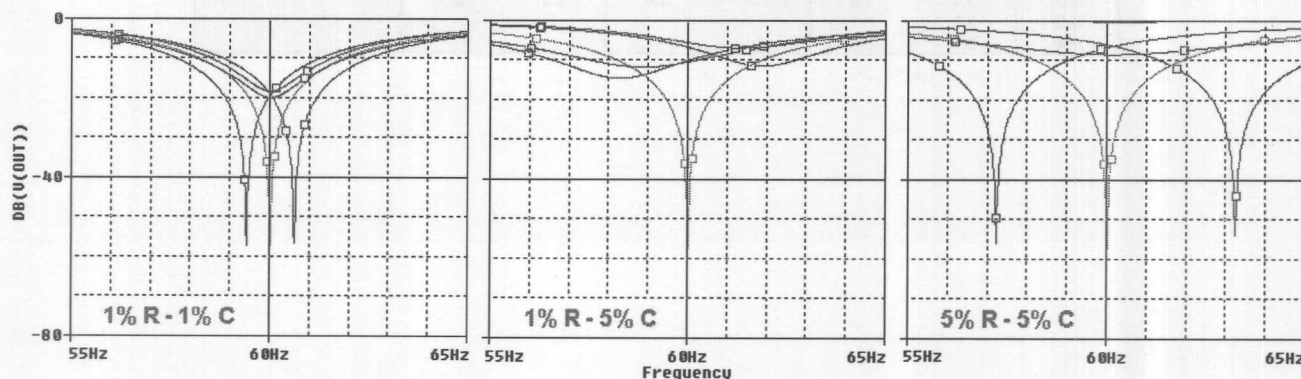


Figure 1. Notch Filter Design With Different Tolerance Components.

Freq	Set 1		Set 2		Set 3		Set 4	
	C	R	C	R	C	R	C	R
	1.2							
1.1	1.2	1.2						
1.2	1.2	1.1						
1.3	6.8	1.8	1.8	6.8				
1.4	4.7	2.4						
1.5	8.2	1.3						
1.6	6.2	1.6	1.6	6.2	3.3	3.0		
1.7	3.9	2.4						
	6.8							
1.9	1.5	5.6						
2.0	3.3	2.4	2.2	3.6				
2.1	4.7	1.6						
2.2	3.3	2.2	2.2	3.3				
2.3	4.3	1.6	1.6	4.3	3.9	1.8	1.8	3.9
2.4	5.1	1.3	1.3	5.1	3.3	2.0	2.2	3.0
2.5	1.5	4.3						
	1.2							
2.7	3.9	1.5	1.5	3.9				
2.8	9.1	6.2	6.2	9.1	4.7	1.2	1.2	4.7
2.9	8.2	6.8	6.8	8.2				
3.0	3.3	1.6	2.2	2.4				
3.1	1.2	4.3						
3.2	3.3	1.5	1.5	3.3				
3.3	2.2	2.2						
3.4	3.9	1.2	1.2	3.9				
3.5	8.2	5.6	5.6	8.2				
3.6	2.2	2.0						
3.7	1.0	4.3						
3.8	8.2	5.1						
	2.7		1.5					
4.0	3.3	1.2	2.2	1.8	1.8	2.2		
4.1	3.9	1.0						
4.2	6.8	5.6						
4.3	3.3	1.1						
4.4	3.3	1.1						
4.5	8.2	4.3						
4.6	6.8	5.1						
	6.8							
4.8	1.5	2.2						
4.9	2.7	1.2	1.8	1.8	1.2	2.7		
5.0	6.8	4.7	4.7	6.8				
5.1	2.4	1.3	1.3	2.4	5.6	5.6		
5.2	2.4	1.3	1.3	2.4	3.3	9.1		
5.3	3.3	9.1						
5.4	8.2	3.6						

	Set 1		Set 2		Set 3		Set 4		Set 5	
	C	R	C	R	C		C		C	R
5.5	1.8	1.6								
5.6	5.6	5.1								
5.7	5.6	5.1								
5.8	9.1	3.0	3.0	9.1	8.2	3.3	3.3	8.2		
5.9	2.7	1.0	1.8	1.5	1.5	1.8	1.0	2.7		
6.0	6.8	3.9	3.9	6.8						
6.1	4.7	5.6								
	1.6	1.6	4.7	5.6						
6.3	1.6	1.6	3.3	7.5						
6.4	3.3	7.5								
6.5	6.8	3.6								
6.6	5.6	4.3								
6.7	4.7	5.1								
6.8	1.8	1.3								
6.9	1.8	1.3								
	3.0	7.5	7.5	3.0	1.5					
7.1	6.8	3.3	3.3	6.8						
7.2	4.7	4.7								
7.3	3.9	5.6	5.6	3.3						
7.4	1.8	1.2	1.2	1.8						
7.5	1.8	1.2	1.2	1.8						
7.6	1.6	1.3	1.3	1.6	3.3	6.2				
7.7	1.6	1.3	1.3	1.6	3.3	6.2				
	6.8	3.0								
7.9	5.6	3.6								
8.0	3.9	5.1								
8.1	8.2	2.4								
8.2	1.5	1.3								
8.3	1.2	1.6								
8.4	1.2	1.6								
8.5	6.2	3.0	3.0	6.2	5.6	3.3	3.3	5.6		
	4.3	4.3	5.6	3.3	3.3					
8.7	4.7	3.9	3.9	4.7						
8.8	8.2	2.2	2.2	8.2						
8.9	7.5	2.4	2.4	7.5	1.8	1.0	1.5	1.2	1.2	1.5
9.0	1.6	1.1	1.1	1.6	1.8	1.0	1.5	1.2	1.2	1.5
9.1	1.6	1.1	1.1	1.6	1.8	1.0	1.5	1.2	1.2	1.5
9.2	1.6	1.1	1.1	1.6	4.7	3.6				
9.3	4.7	3.6								
	4.7	3.6								
9.5	2.7	6.2								
9.6	1.5	1.1								
9.7	8.2	2.0								
9.8	6.8	2.4								
9.9	1.0	1.6								

Table 3. E24 Combinations

6 Conclusion

The decision about which components to purchase to cover the needs of a filter design lab ultimately involves a tradeoff between budget and the need to design all combination of frequency. There are some absolutes, however, such as the need to procure 1% resistors if accurate frequency response is needed, and the need to purchase 1% capacitors if high Q notch or bandpass filters are to be designed.

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More Filter Design on a Budget

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ABSTRACT

This document describes filter design from the standpoint of cost. Filter design techniques that require the fewest possible op amps and passive components are described. Six types of filters are described—low pass, high pass, narrow bandpass, wide bandpass, notch, and band reject.

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1 Introduction

Why *filter design on a budget*? The answer is self-evident—fewer op amps are less expensive, take up less space on a PC board. Fewer passive components also means less space on a PC board, less parts to stock, less assembly and less test time. It seems that every reference on filter design that the author encountered was written by academics whose love for mathematical derivation was more important than considerations of putting a real design into production. Naturally, all filter topologies are presented in their works without an interpretation of *what is best*. In some applications, it might make sense to use topologies that use three op amps to implement only two poles—perhaps the end equipment is expensive and a few extra components are not a concern. This author suspects that the vast majority of applications are constrained in cost and PC-board space. A telephone handset that uses only one op amp and four passive components to filter speech is less expensive and smaller than one that uses three op amps and 10 passive components for the exact same filter response. These are the new realities of analog design.

The first article in this series – *Filter Design on a Budget* (reference 1) concentrated on passive components—just those values which are really needed to implement a filter with the desired frequency response. This article concentrates on the actual filter topologies. It answers the question, *Without compromising response in any way, how can a filter be implemented using the minimum number of op amps and passive components?* It focuses exclusively on double pole Butterworth response filters, although other filter response characteristics can be accomplished, using the proper design techniques.

2 Low Pass Filter

A low pass filter is used to eliminate high frequency harmonics from an analog waveform. It has a response that extends from dc to a cutoff frequency, which is defined as the point at which the amplitude has declined 70.7% (or 3 dB) from its initial value at low frequencies. The response of a Butterworth double pole low pass filter is shown in Figure 1. After the initial 3 dB attenuation (shown at the red marker), the response at a frequency ten times higher (shown by the blue marker) is down 40 dB (a one hundred times reduction).

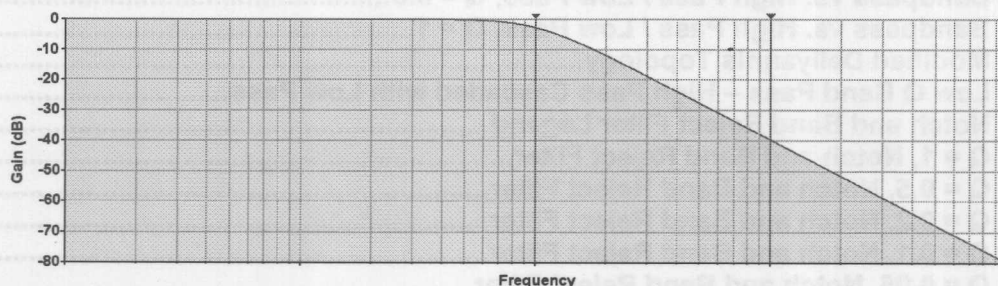


Figure 1. Low Pass Filter Response

In practice, low pass filter response degrades close to dc. Most op amps exhibit a *pink noise* characteristic at low frequencies, which eventually makes the op amp very noisy at very low frequencies (milli- or micro- Hertz). In addition, single-supply op amp circuits employ dc blocking capacitors, which introduce a one-pole high pass characteristic to the response. The designer can place this high pass pole as low in frequency as desired, however.

Truism: There is no such thing as an ac-coupled single-supply active low pass filter. They are bandpass filters with a low frequency cutoff determined by the selection of the coupling capacitor.

There are two very good double pole low pass topologies—Sallen-Key and Multiple Feedback (MFB). Sallen-Key, as shown in Figure 2 is also available in a version with gain, but there is little advantage to it—it adds two additional resistors. The MFB topology can be used for gains of more than one.

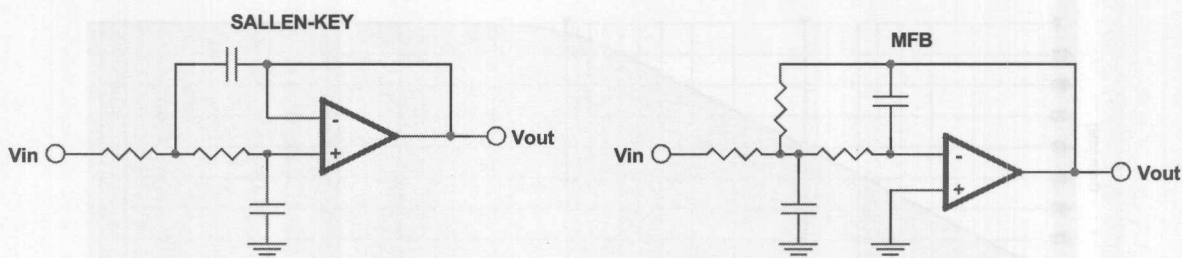


Figure 2. Low Pass Filter

Component count:

Op amp:	1
Capacitor:	2
Resistor:	2 to 3, depending on topology selected

Single Supply Modification:

Single supply modification is easily accomplished for the MFB topology by moving ground connections to half supply, and ac-coupling. It is difficult for the Sallen-Key topology—it uses two additional resistors to create a virtual ground at the input. It is better to use MFB for single supply low pass applications.

Fully-Differential Modification:

Fully-differential modification is easily accomplished by duplicating the feedback path for the MFB topology. It is not possible for Sallen-Key topology.

Design Procedure:

Design procedure is too complex for inclusion here—refer to a textbook on the topic.

Limitations:

The Sallen-Key topology shown above is limited to unity gain. Although it is possible to use two additional resistors with the Sallen-Key topology to provide gain, there is no advantage to doing so. The MFB topology can accomplish the same thing with one less resistor.

3 High Pass Filter

A high pass filter is used to eliminate low frequency harmonics from an analog waveform. It has a response that extends down from infinity to a cutoff frequency, which is defined as the point at which the amplitude has declined 70.7% (or 3 dB) from its initial value. The response of a Butterworth double pole high pass filter is shown in Figure 3. The 3 dB attenuation frequency is shown at the red marker. The response at one-tenth the –3 dB frequency (shown by the blue marker) will be down 40 dB (a one hundred times reduction).

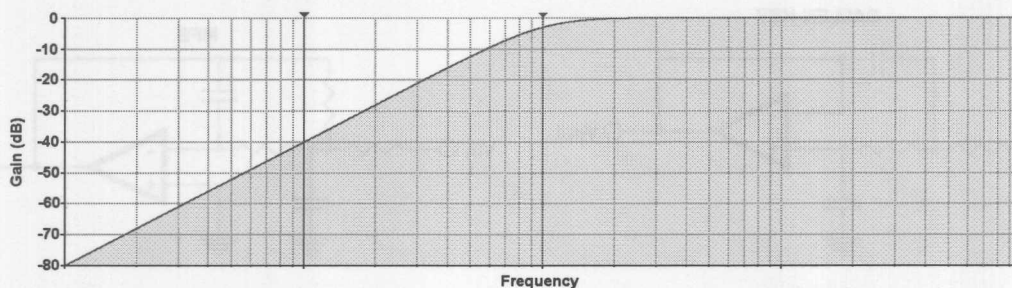


Figure 3. High Pass Filter Response

Figure 3 implies that the filter can pass energy out to infinity. In practice however, high pass filter response does not extend to infinity. Op amps have an ultimate bandwidth limitation, which is the point at which the closed loop response of the op amp intersects the open loop response. This is the gain bandwidth limitation of the op amp, and the response rolls off at –20 dB per decade above this limit, which gives a one-pole low pass response.

Truism: There is no such thing as an active high pass filter. They are bandpass filters with a high frequency cutoff determined by the selection of op amp and gain.

There are two very good double pole high pass topologies—Sallen-Key and Multiple Feedback (MFB). Sallen-Key as shown in Figure 4 is also available in a version with gain, but there is little advantage—it adds two additional resistors. The MFB topology can be used for gains more than one.

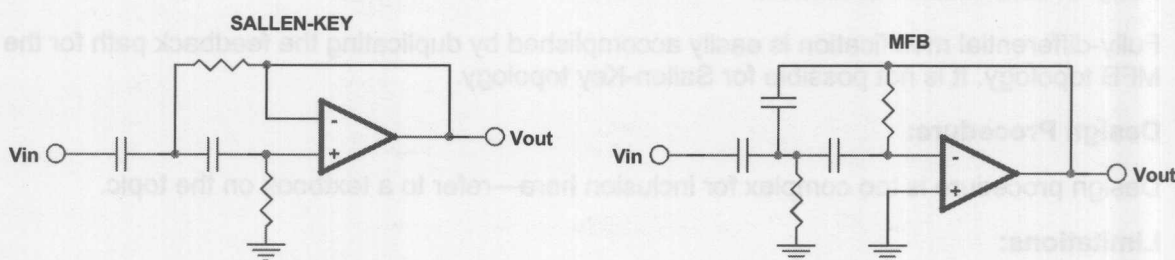


Figure 4. High Pass Filter

Component count:

Op amp: 1

Capacitor: 2

Resistor: 2 to 3, depending on topology selected

Single Supply Modification:

Single supply modification is easily accomplished for either topology. Accomplished by moving ground connections to half supply. AC-coupling is not required at the input; capacitors associated with the high pass topology act to isolate dc potential.

Fully-Differential Modification:

Fully-differential modification is easily accomplished by duplicating the feedback path for the MFB topology. It is not possible for Sallen-Key.

Design Procedure:

Design procedure is too complex for inclusion here – refer to a textbook on the topic.

Limitations:

The Sallen-Key topology shown above is limited to unity gain. Although it is possible to use two additional resistors with the Sallen-Key topology to provide gain, there is no advantage to doing so. The MFB topology can accomplish the same thing with one less resistor.

4 Bandpass Filters

Bandpass filters are used for everything from tone detection to passing a broad range of frequencies. Depending on the bandwidth requirements, these tasks can require completely different design approaches. This application note uses the terms *narrow bandpass* and *wide bandpass*.

The Sallen-Key and MFB topologies have bandpass variations. They place different types of components in impedance locations in a topology. For example, a resistor may be changed to a capacitor in the MFB topology. This serves to take one of the poles of a double pole low pass / high pass variation, and convert it to the other type. A two-pole low pass filter, for example, has one of its poles changed to a high pass pole, leaving one high pass pole and one low pass pole. Similarly, a high pass filter is converted to a bandpass by taking one high pass pole and converting it to low pass, leaving one high pass and one low pass pole.

This *one pole* response characteristic is not the end of the story. From the preceding discussion, the designer would expect only 20 dB per decade rolloff in the stop bands regardless of the Q (sharpness) of the filter. But that is not the case—the transfer function of a bandpass filter forces the response to take whatever slope is necessary to satisfy the gain at the center frequency and the –3 dB points. The slope of the response of high Q bandpass filters can be quite steep near the center frequency. All bandpass filters, however, revert to 20 dB per decade rolloff characteristic away from the center frequency. As the Q becomes lower, the response begins to look more and more like a single pole low pass filter on the low end of the pass band, and a single pole high pass filter on the high end of the pass band.

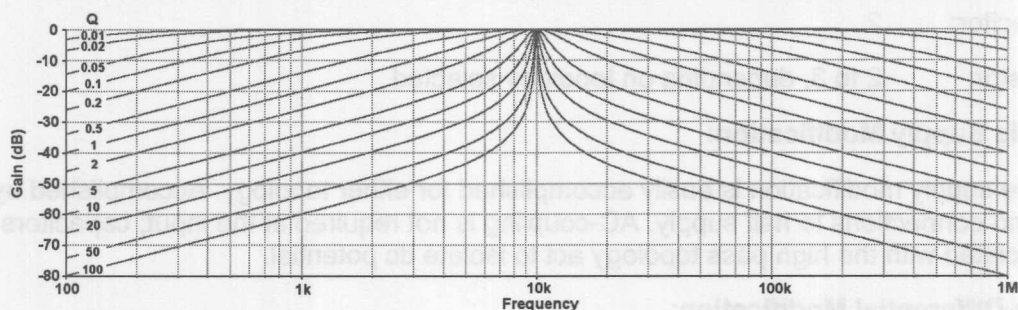


Figure 5. Bandpass Filter Q Comparison

This leads to a question—is it more advantageous to implement a wide band pass by implementing a low pass filter and a high pass filter? If a designer uses cascaded bandpass stages, the best that can be obtained is additional first order rolloff on the low and high end. If the designer concentrates separately on the low and high ends of the band, the result is far superior. Often times, the requirement for rejection on one or the other end of the band is different from the requirement at the other. It may be very stringent for the high frequency end, but the low end of the band may only have the requirement to reject dc (ac-couple). Therefore, it is better to implement low Q bandpass filters as cascaded high pass and low pass filters. The only tricky part for the designer is determining at what point the tradeoff occurs.

The figures to follow show a progression of Q values from 0.1 to 1. The bandpass implementation is shown in red and the cascaded high pass and low pass implementation in blue. Different regions have different shading in the figures, according to the legend in Figure 6:

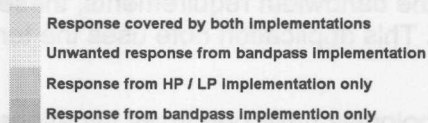


Figure 6. Legend for Bandpass Filter Responses

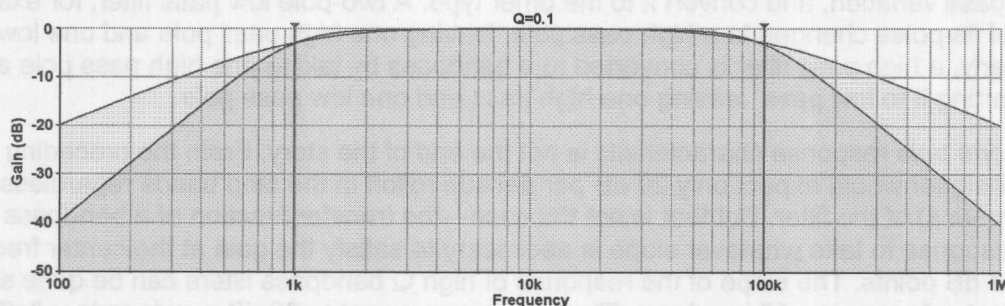


Figure 7. Bandpass vs High Pass / Low Pass, Q = 0.1

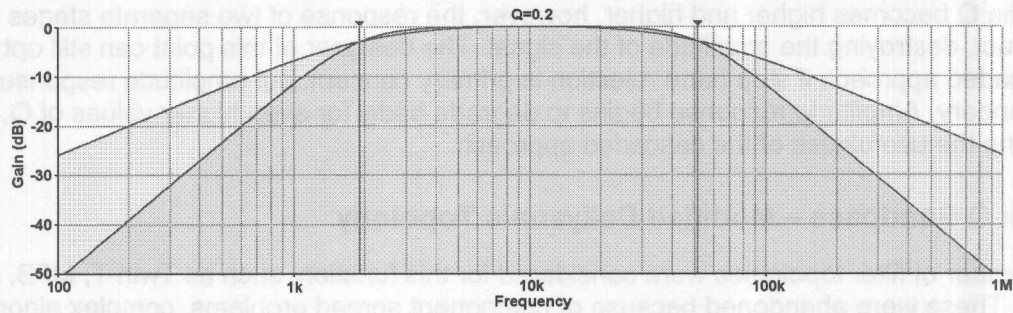


Figure 8. Bandpass vs High Pass / Low Pass, $Q = 0.2$

Clearly, for Q values of 0.1 (and below), and 0.2, the best implementation is high pass cascaded with low pass. The yellow regions correspond to a large amount of energy in the stop bands that is not rejected with a band pass filter. In the pass band, the cascaded approach is also clearly superior, because there is a wider region in the passband where response is flat.

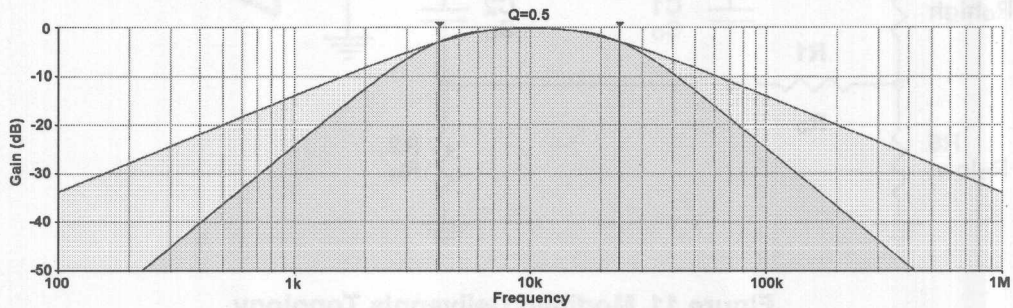


Figure 9. Bandpass vs High Pass / Low Pass, $Q = 0.5$

The two implementations have almost an identical pass band response for a Q of 0.5. The designer is presented with a choice—use a bandpass filter (which can be implemented with a single op amp) to save money, or use a cascaded approach that has better rejection in the stop bands.

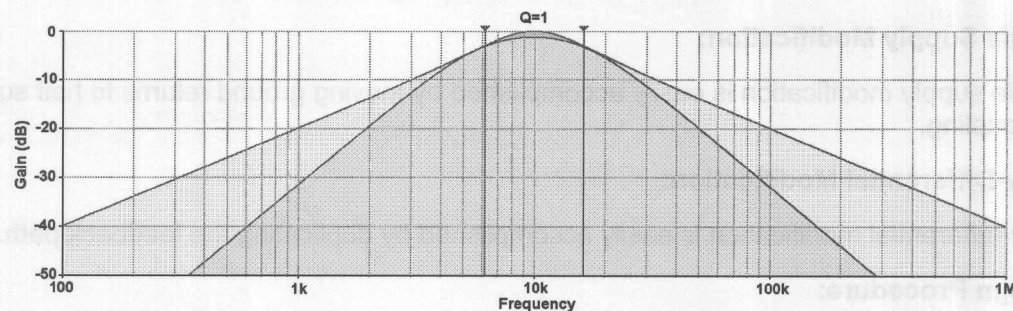


Figure 10. Bandpass vs High Pass / Low Pass, $Q = 1$

As the Q becomes higher and higher, however, the response of two separate stages begins to interact, destroying the amplitude of the signal. The designer at this point can still opt for the cascaded approach if stop band rejection is primary concern, but amplitude response is secondary. Amplitude response begins to degrade badly for even higher values of Q, however, ending the usefulness of the cascaded approach.

4.1 High Q Bandpass – Modified Deliyannis Topology

A number of filter topologies were considered for this function, such as Twin T, MFB, and Sallen-Key. These were abandoned because of component spread problems, complex algorithms, or other problems. Even the Deliyannis topology is not perfect, but it seems to be the best of the single op-amp bandpass topologies. When modified as shown in Figure 7, it is easy to use.

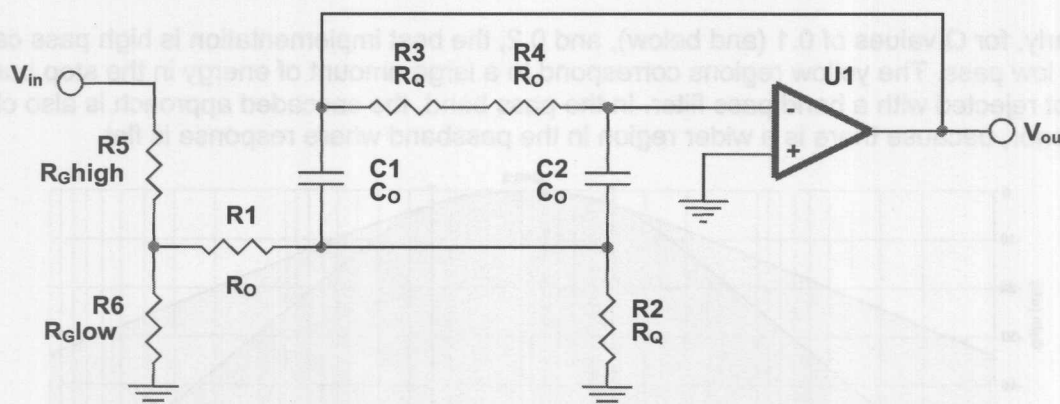


Figure 11. Modified Deliyannis Topology

Component count:

Op amp: 1

Capacitor: 2

Resistor: 3 (if R3 and R4 are combined) to 5, depending on gain and Q needed

Single Supply Modification:

Single supply modification is easily accomplished by moving ground returns to half supply and ac coupling.

Fully-Differential Modification:

Fully-differential modification is easily accomplished by duplicating the feedback path.

Design Procedure:

- The center frequency is determined by the relation:

$$f_o = \frac{1}{2\pi R_o C_o} \quad (1)$$

Where:

$$R1 = R4 = R_o$$

$$C1 = C2 = C_o.$$

- Gain and Q are both determined by the expression:

$$\frac{R3 + R4}{2 * R1} = \frac{V_{out}}{V_{in}} = Q \quad (2)$$

Where:

$$n * R3 = R_o = \frac{1}{n} * R2 \quad (3)$$

If R3 is doubled, R2 must be halved and vice versa. If one is tripled, the other must be one third, etc. R2 and R3 must always be related in this way. Otherwise, the center frequency and other circuit characteristics are changed.

- Because Gain and Q are linked together, gain resistors R5 and R6 can be used as a voltage divider to reduce the input level and compensate for this effect. When Gain and Q approach one, short R5 and open R6.

Watch the gain bandwidth product of the op amp carefully for high values of Q. Allow at least 40 dB of safety margin above the peak at the resonant frequency. Also, use an op amp with a high slew rate.

If R1 = R2 = R3 = R4, then Q and Gain are both equal to one.

Limitations:

The circuit cannot be used below a gain and Q of 0.5, because at these values, R3 has to be zero and R2 must be infinite (open). There is no way to boost the gain at Q values less than one, other than to use a separate gain stage. This increases the op amp count by one and the resistor count by two.

4.2 Low Q Bandpass – Cascaded High Pass / Low Pass Topology

The best way of implementing a low Q bandpass filter is to cascade a high pass filter and a low pass filter (in that order). It is preferable to make the high pass stage first, because high frequency noise generated by it can be attenuated in the final low-pass stage.

There are two common single op amp topologies – Sallen-Key and MFB.

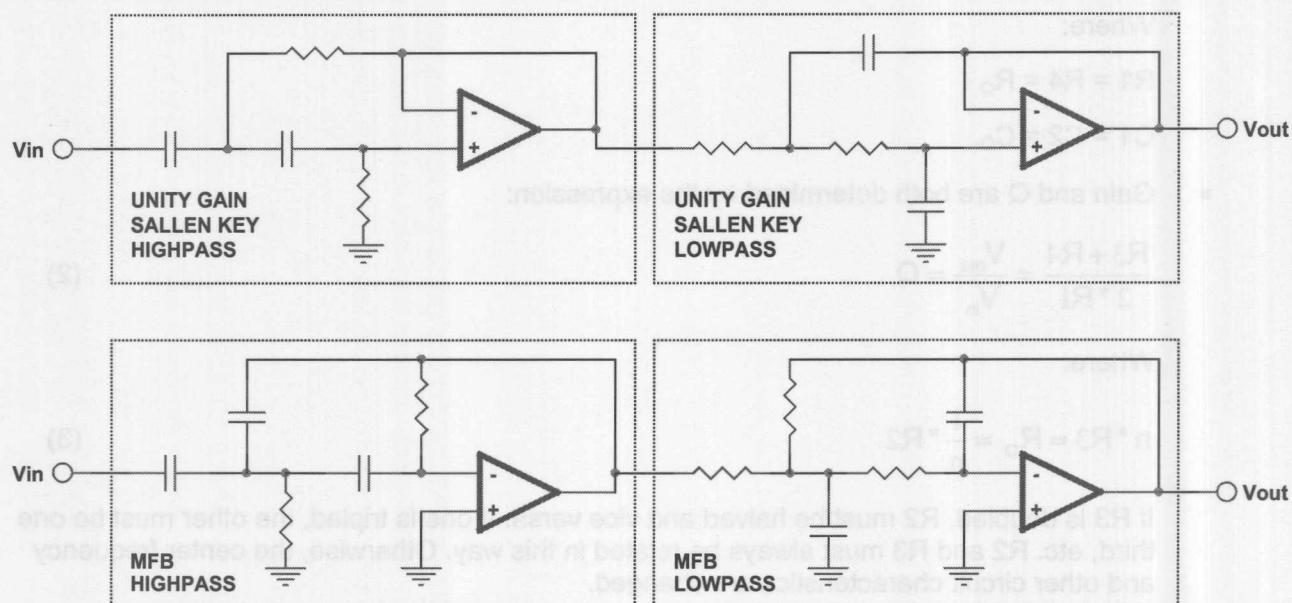


Figure 12. Low Q Band Pass – High Pass Cascaded With Low Pass

Component count:

Op amp:	2
Capacitor:	4 to 5, depending on filter topology selected for low pass and high pass
Resistor:	4 to 5, depending on filter topology selected for low pass and high pass

Single Supply Modification:

Single supply modification is easily accomplished for both implementations by moving ground returns to half supply and ac-coupling.

Fully-Differential Modification:

Fully-Differential modification is possible for the MFB implementation only—by duplicating the feedback path.

Design Procedure:

Design procedure is beyond the scope of this application note, but covered in numerous textbooks and application notes. Design a high pass filter for the lower end of the range, and a low pass filter for the upper end of the range.

Limitations:

- Complex design procedure
- Sallen-Key approach is limited to unity gain with four passive components.

5 Notch and Band Reject Filters

A notch filter is primarily used to reject a single frequency, while a band reject filter is designed to reject a range of frequencies. There are implementations similar to the bandpass case—a true notch corresponding to a narrow bandpass, and a band reject corresponding to a wide band pass.

The depth of the notch for notch filters is largely independent of the Q . Any appearance to the contrary in the figures to follow is an accident of the number of samples used to generate the plot. Q affects the bandwidth of where the -3 dB points lie, which results in a gradually more *washed out* appearance of the notch filter response (blue trace) as shown in the sequence from Figure 14 to Figure 20. Any of the notch filter Q values shown give excellent rejection of the center frequency. If a band of frequencies is to be rejected, however, a notch filter is not the most efficient way to do it. As the Q becomes lower and lower, a lot of excess energy is passed, as shown in the yellow areas of Figure 14 through Figure 20.

The cascaded implementation that worked well for wide bandpass applications cannot be used for wide notch (band reject) filters. This is because the response characteristics have to overlap, or everything becomes attenuated. The only technique that forms a band reject filter is a summed low pass and high pass stage. The response of this configuration is shown in red in Figures 14 to 20. At a Q of 1, it only has a rejection of 7 dB, and is almost useless. It begins to have better rejection as Q values are decreased. At a Q value of 0.05, rejection is over 40 dB—making the summed low pass and high pass implementation clearly superior for band rejection filter. The pink area, however, shows energy near the center frequency that only the notch filter can reject. The designer must decide whether the center frequency rejection is of prime importance, or whether it is better to reject a band of frequencies.

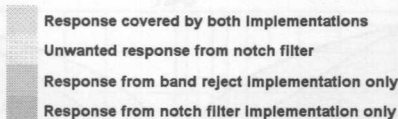


Figure 13. Notch and Band Reject Filter Legend

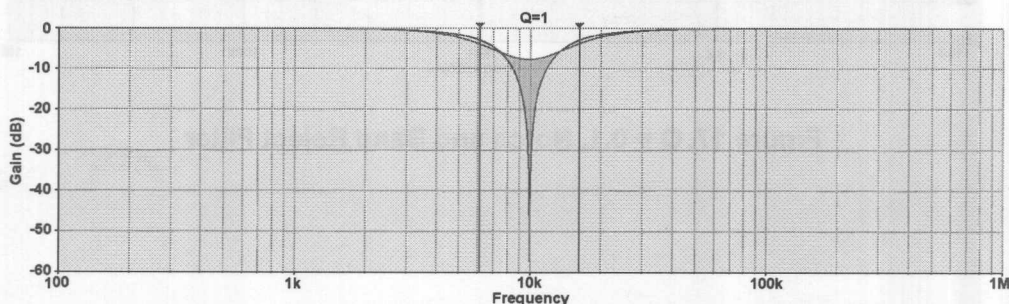
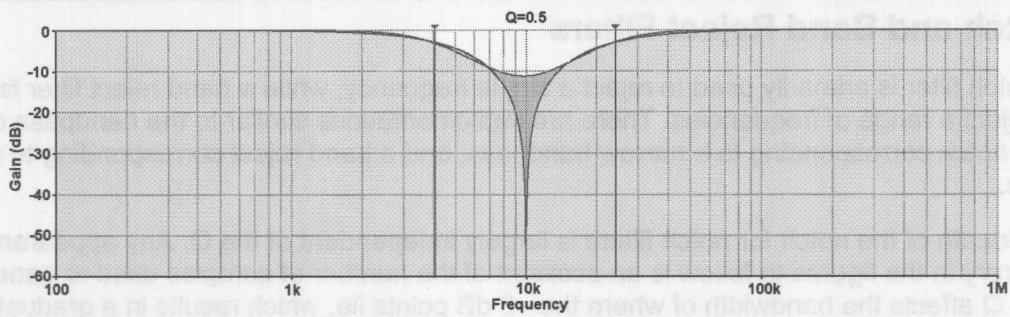
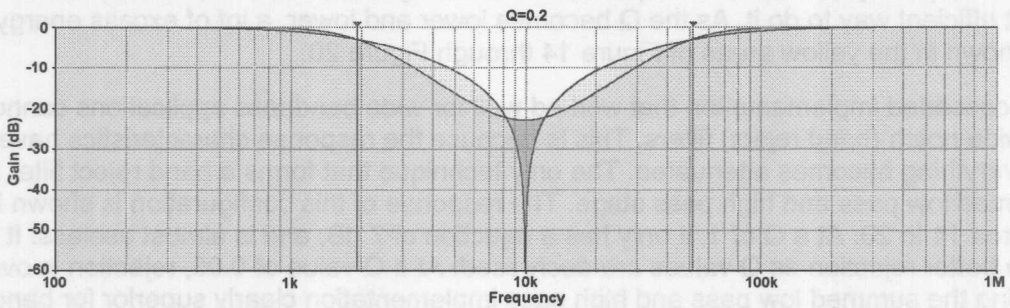
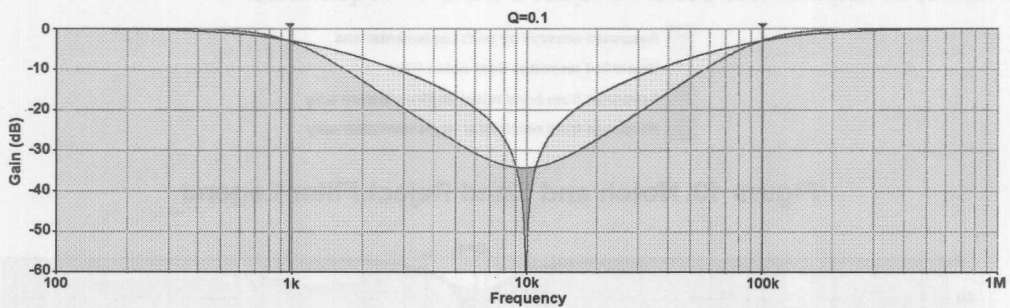


Figure 14. $Q = 1$, Notch and Band Reject Filter

**Figure 15. Q = 0.5, Notch and Band Reject Filter****Figure 16. Q = 0.2, Notch and Band Reject Filter****Figure 17. Q = 0.1, Notch and Band Reject Filter**

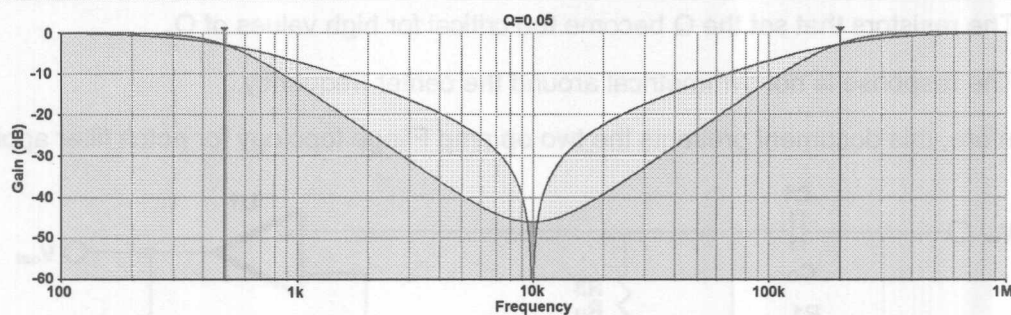


Figure 18. $Q = 0.05$, Notch and Band Reject Filter

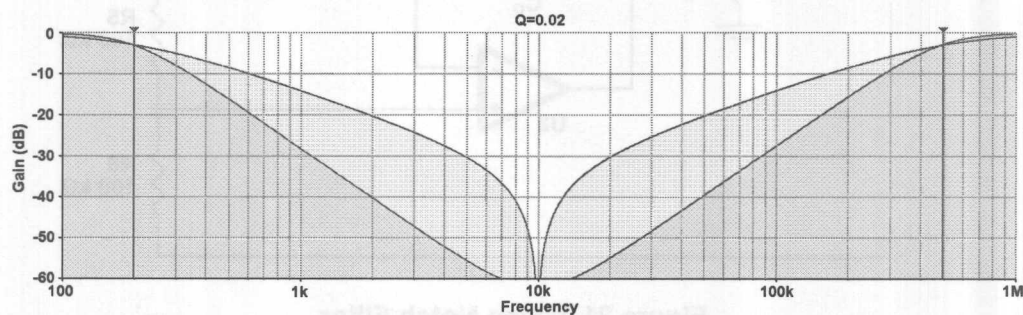


Figure 19. $Q = 0.02$, Notch and Band Reject Filter

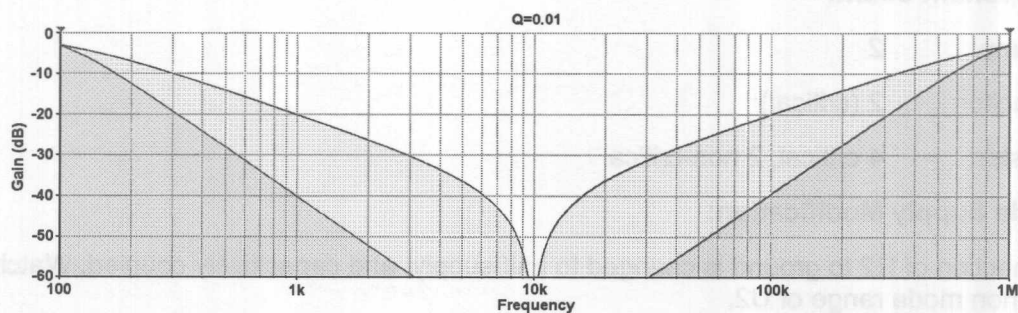


Figure 20. $Q = 0.01$, Notch and Band Reject Filter

5.1 Notch Filter – Fliege Topology

A number of notch topologies exist. It is very easy to accomplish a good notch filter with three op amps, and not so easy to implement a notch in one or two op amps. This document assumes that notch filters have unity gain, which simplifies things somewhat.

The Sallen-Key and Twin T notch topologies were considered, but reluctantly abandoned as being impractical for one or more of the following reasons:

- There is not a good algorithm that describes the relationship between resistor value and Q .

- The resistors that set the Q become too critical for high values of Q.
- The response is not symmetrical around the center frequency.

Therefore, this document presents the two op amp Fliege topology for notch filter applications.

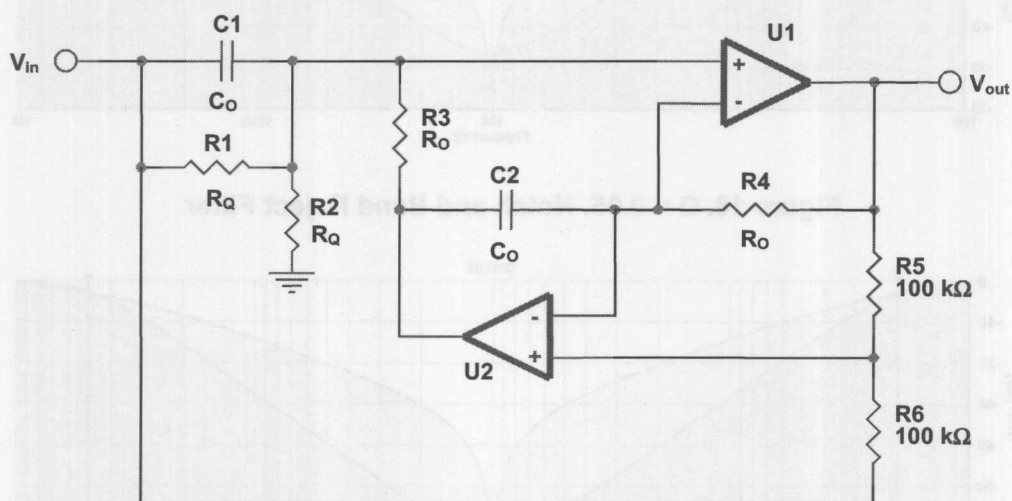


Figure 21. Fliege Notch Filter

Component count:

Op amp: 2
 Capacitor: 2 (critical)
 Resistor: 4 critical, 2 non-critical

Single Supply Modification:

Connection of R2 to ground is changed to half supply, and capacitively coupled. Watch the common mode range of U2.

Fully-Differential Modification:

Fully-differential modification is not possible.

Design Procedure:

- The center frequency is determined by the relation:

$$f_o = \frac{1}{2\pi R_o C_o} \quad (4)$$

Where:

$$R3 = R4 = R_o$$

$$C1 = C2 = C_o.$$

- Q is determined by the expression:

$$R_Q = 2 * Q * R_o \quad (5)$$

R5 and R6 are non-critical, but should be the same value.

Limitations:

- Limited to unity gain

5.2 Band Rejection Filter—Summed High Pass / Low Pass Topology

Band rejection filters are used when a relatively wide band of frequencies need to be rejected. Possible situations would be a switching power supply conversion frequency that changes with load, or harmonics from an unlocked phase locked loop circuit.

Figure 22 shows the implementation of a band rejection filter using summed low pass and high pass stages.

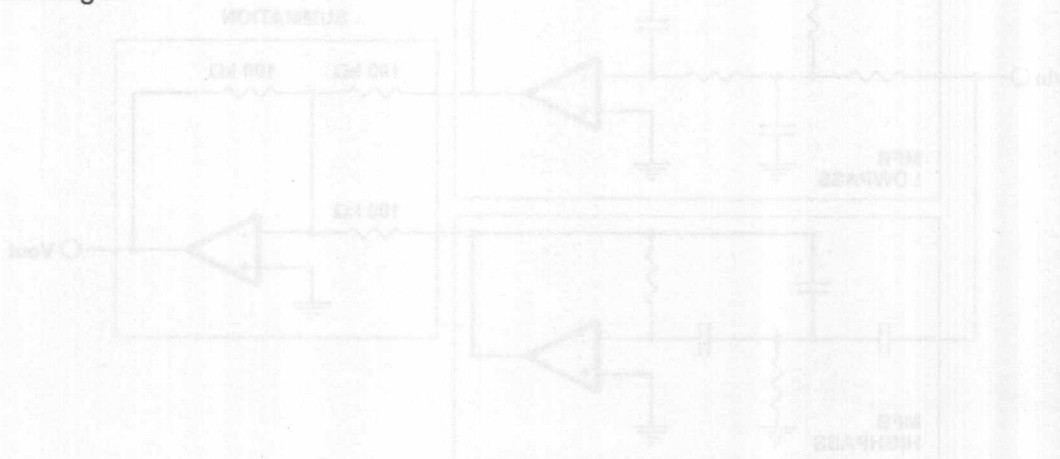


Figure 22. Band Rejection Filter Implementation

It should be pointed out that the op amp required by the summation stage may already be present—a buffer op amp for an analog-to-digital converter, for example. So it may not be a significant increase in cost.

Component count:

Op amp: 3

Capacitor: 4 to 5, depending on filter topology selected for low pass and high pass

Resistor: 6 to 8, depending on filter topology selected for low pass and high pass

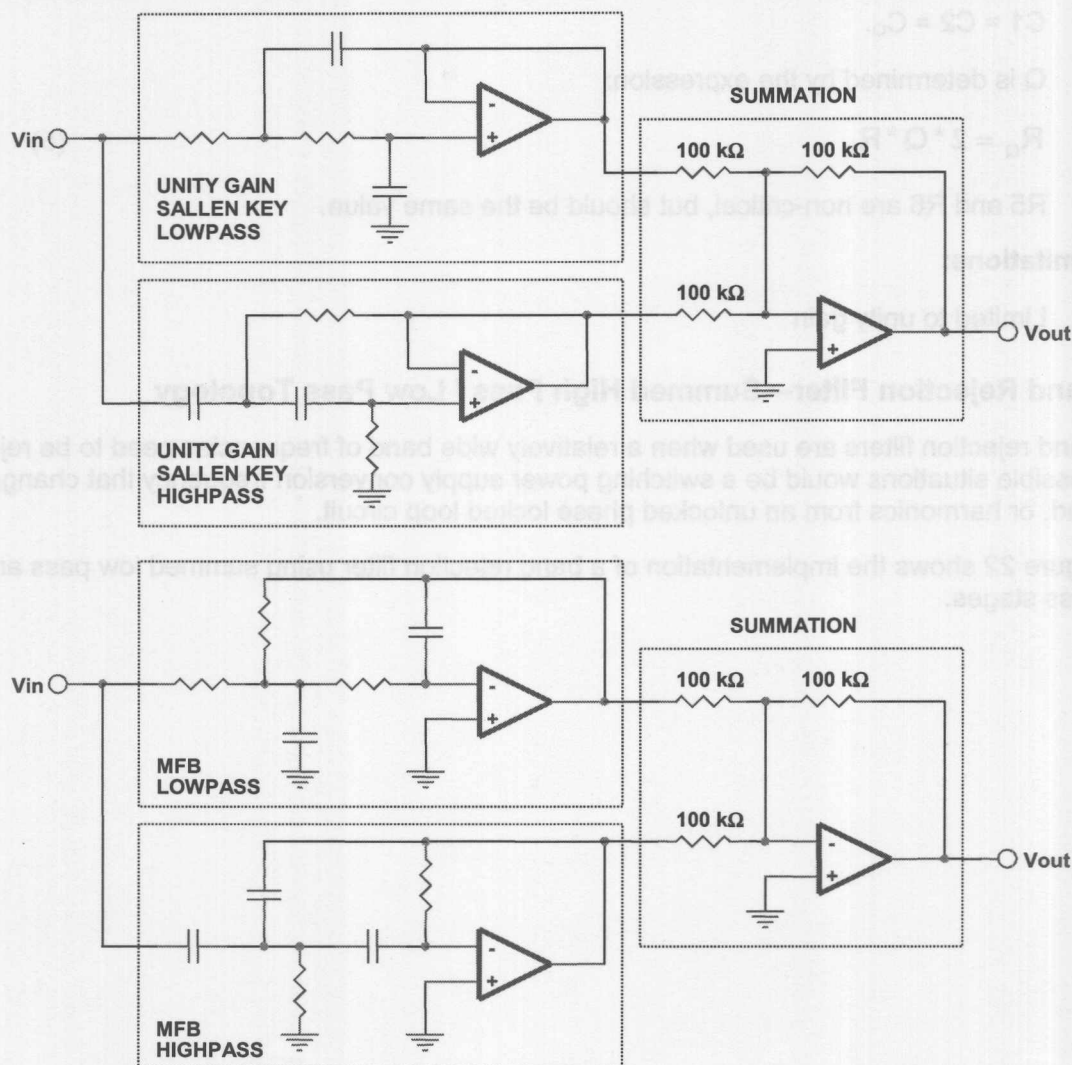


Figure 22. Band Reject Filter Implementations

It should be pointed out that the op amp required by the summation stage may already be present—a buffer op amp for an analog to digital converter, for example. So it may not be a significant increase in cost.

Component count:

Op amp:	3
Capacitor:	4 to 5, depending on filter topology selected for low pass and high pass
Resistor:	6 to 8, depending on filter topology selected for low pass and high pass

Single Supply Modification:

Single supply modification is easily accomplished for both implementations by moving ground returns to half supply and ac-coupling.

Fully-Differential Modification:

Fully-differential modification is possible for the MFB implementation only—by duplicating the feedback path.

Design Procedure:

Design procedure is beyond the scope of this application note, but covered in numerous textbooks and application notes. Design a high pass filter for the lower end of the range, and a low pass filter for the upper end of the range.

Limitations:

- Complex design procedure
- Sallen-Key is limited to unity gain as shown above.

References

1. *Filter Design on a Budget*, Texas Instruments SLOA065

Appendix A—Summary of Filter Characteristics

Table 1. Cost of Implementation

Desired Function	Topology	Op Amp	C	R	Q	Limitations
Low Pass	Sallen-Key	1	2	2		Unity gain
	MFB	1	2	3		
High Pass	Sallen-Key	1	2	2		Unity gain
	MFB	1	3	2		
Narrow Bandpass	Deliyannis	1	2	3 to 6	0.5 to ∞	Gain and Q Interact
Wide Bandpass	Cascaded HP LP SK	2	4	4	< 0.5	Unity gain
	Cascaded HP LP MFB	2	5	5	< 0.5	
Notch	Fliege	2	2	4	0.05 to ∞	Unity gain
Band Reject	Summed HP LP SK	3	4	6	< 0.5	Unity gain
	Summed HP LP MFB	3	5	8	< 0.5	

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